Editors' Suggestion

Nonequilibrium Prethermal States in a Two-Dimensional Photon Fluid

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We report on the observation of a prethermal state in a nonequilibrium, two-dimensional fluid of light. Direct measurements of the first order coherence function of the fluid reveal the dynamical emergence of algebraic correlations, a quasi-steady-state with properties close to those of thermal superfluids. By a controlled increase of the fluctuations, we observe a crossover from algebraic to short-range (exponential) correlations. We interpret this phenomenon as a nonequilibrium precursor of the Kosterlitz-Thouless transition.

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Thermalization is the dynamical process by which any subpart of a many-body system evolves toward a thermal equilibrium state that maximizes its entropy. While a general description of how thermal equilibrium establishes in quantum systems remains elusive, a variety of scenarios have been identified [1,2]. Examples include the nonequilibrium dynamics of near-integrable systems [3], the relaxation toward thermalization in correlated gases [4], the spontaneous emergence of universal scaling laws [5–7] following a quantum quench, and even the absence of thermalization in the presence of disorder [8]. In nonequilibrium many-body systems, the phenomenon of prethermalization plays a peculiar role [9–12]. Prethermalization describes a significant slowing down of the dynamics, where the system initially relaxes toward a quasisteady, long-lived state after a perturbation. The establishment of thermal equilibrium eventually occurs over a much longer timescale. In a prethermal state, the system retains a partial memory of its initial conditions, while showing a strong resemblance to its true thermal equilibrium state [9–17]. Experimentally, most studies of this phenomenon have been conducted in ultracold atomic gases: relaxation and prethermalization have been observed in one-dimensional (1D) Bose gases [18–21], where the dynamical emergence of prethermal states arises because the system is close to integrability. Recently, signatures of prethermal states were also identified in unitary Bose gases [22].

In parallel, fluids of light in the propagating geometry [23] have emerged as a complementary platform to study two-dimensional (2D) Bose gases, with the observation of Bogoliubov-like dispersion [24–27], signatures of photon condensation [28,29] and spontaneous nucleation of vortices in a lattice [30]. This platform relies on the formal analogy between a laser field propagating through a nonlinear medium and the temporal evolution of a 2D quantum fluid. In this propagating geometry, the initial state can be engineered at will using wavefront shaping techniques, as illustrated by observations of dispersive shock waves [31–34]. Moreover, upon entering the nonlinear medium, the beam experiences a sudden change of refractive index, which effectively reproduces the dynamics of a Bose gas after an interaction quench [13,15,35]. This makes it a natural system for exploring nonequilibrium physics.

In this Letter, we study the dynamical emergence of a prethermal state in a nonequilibrium, 2D fluid of light after an interaction quench. We first present direct measurements of the fluid's first-order correlation function that reveal the spontaneous emergence of long-range algebraic correlations spreading within a light cone, a characteristic signature of prethermalization bearing strong similarities with 2D thermal superfluids [36–41]. We then provide a detailed experimental characterization of the algebraic order and find an agreement with recent theoretical predictions [15]. Finally, by a controlled increase of the fluid fluctuations, we unveil a crossover from algebraic to short-range (exponential) correlations, analogous to the celebrated Kosterlitz-Thouless transition observed at thermal equilibrium.

Experimentally, our fluid of light is created by letting a laser beam propagate through a 10 mm-long vapor cell of ^{87}Rb heated to a temperature of 150 °C. Effective photon-photon interactions are achieved by tuning the laser close to resonance (detuned by -1.5 ± 0.1 GHz) with respect to the $F = 2 \rightarrow F'$ transition of the D2 line at $\lambda_0 = 2\pi/k_0 =$ 780 nm, as described in [24,42]. Under these conditions, the vapor is self-defocusing, corresponding to repulsive photon-photon interactions. As proposed in [15], we carefully prepare an initial state consisting of weak random speckle field $\psi_s(\boldsymbol{r})$ superimposed on a more intense laser beam having a wide Gaussian profile (waist $w_0 =$ 1.8 mm) and denoted $I_t(r) = I_0 \exp(-2r^2/w_0^2)$. The optical field impinging on the cell is of the form

$\Psi(\boldsymbol{r}, z=0) = \sqrt{I_t(r)}[1+\epsilon\psi_s(\boldsymbol{r})]/\sqrt{1+\epsilon^2}$, where $\epsilon \ll 1$ is a dimensionless parameter quantifying the fluctuations and controlled via spatial light modulator (SLM) patterns. The key idea here is that the background field, being close to a plane wave, is almost a stationary state of the problem. The weak speckle fluctuations thus mainly drive the thermalization process. For $\epsilon \ll 1$, however, the latter becomes slow and leaves room for a prethermal stage where the dynamics becomes governed by independent phonons of long lifetime [15]. Notice that the chosen initial state can be seen as the optical analogue of a 2D trapped Bose-Einstein condensate [of spatial profile $I_t(r)$ and condensate fraction $1/(1+\epsilon^2)$], on top of which small thermal fluctuations (here described by the random speckle) are present [43].

At the cell entrance, the photon fluid effectively experiences an interaction quench due to the nonlinear index change. The cell exit plane is imaged on a camera after propagating within a balanced Mach-Zehnder interferometer. The images of both arms are inverted with the help of two Dove prisms in a perpendicular configuration [44,45], see Fig. 1(a). The coherence function, $g^{(1)}$, is obtained by Fourier filtering the interference contribution, and averaging the field at the cell exit $\Psi(\boldsymbol{r}, z=L)$ over 2000 speckle realizations:

$$g^{(1)}(\Delta\boldsymbol{r} = 2\boldsymbol{r}) = \langle \Psi^*(\boldsymbol{r}, L)\Psi(-\boldsymbol{r}, L)\rangle, \tag{1}$$

where $\langle\cdots\rangle$ refers to ensemble averaging [see Figs. 1(c)–1(f) and [46] for details].

Typical $g^{(1)}$ measurements are presented in Fig. 1(g). We show the raw data (light curves) obtained for a given Δr, their azimuthal average performed by exploiting the statistical rotational invariance of $g^{(1)}$ and, finally, the data after background residual inhomogeneity correction in dark colors (see Ref. [46]). While, at $z=0$, the coherence function nearly coincides with the background profile $I_t(r)$ due to small initial fluctuations, at $z=L$ one observes a characteristic structure (within the background envelope) where $g^{(1)}$ first decays algebraically and then saturates at a constant value forming a plateau. A low-energy effective theory for the propagation of excitations in the vapor, assuming a homogeneous background laser, provides the dynamical scenario of 2D prethermalization, sketched in Fig. 1(b) [15]. After a short propagation distance $z \sim 1/(2gI_0)$ after the quench, the initial weak speckle fluctuations are amplified, and a quasi-long-range order emerges spontaneously within a light cone of radius $\Delta r \sim 2c_s z$, where $c_s = \sqrt{gI_0/k_0}$ is the speed of sound in the fluid. The position of the light cone can be calculated from the

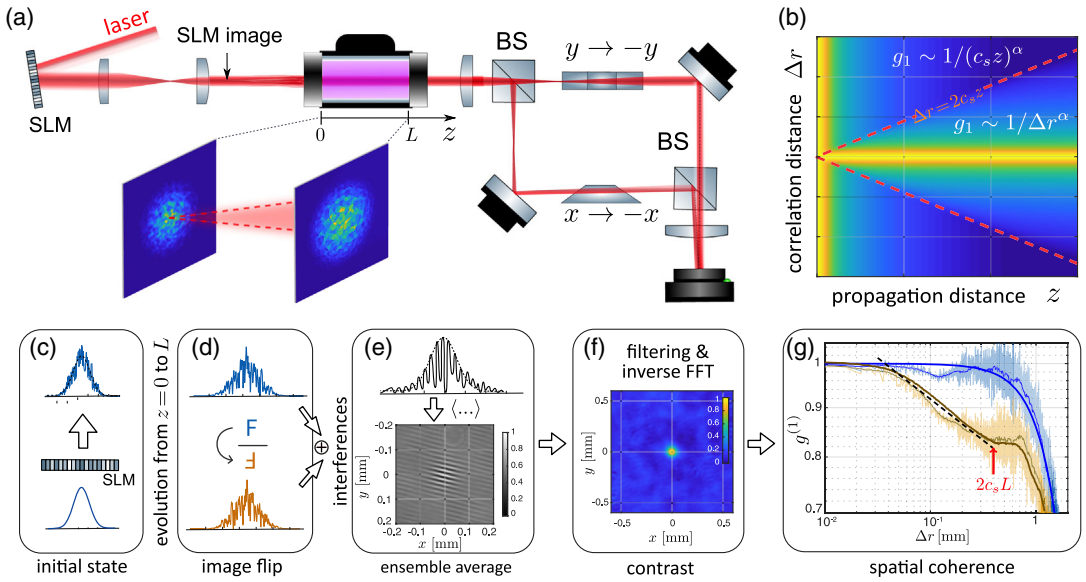

FIG. 1. Measurement of the coherence function demonstrating light-cone spreading of algebraic correlations in a 2D photon fluid. (a) Experimental setup. (b) Scenario of prethermalization in 2D. Upon propagation, the initial short-range speckle fluctuations are amplified and exhibit algebraic correlations associated with long-lived phononic excitations and spreading within a light cone of boundary $\Delta r = 2c_s L$. Out of the light cone, g_1 is independent of $\Delta\boldsymbol{r}$, reproducing the initial state long-range coherence (c) The initial state is prepared using a SLM with a random phase mask superimposed on a Gaussian laser beam. This results in a weakly fluctuating field that propagates in a hot ^{87}Rb vapor cell of length L. (d) After propagation in the cell, the beam is split within a Mach-Zehnder interferometer and flipped using two Dove prisms. (e) The two inverted copies interfere and the fringe visibility is recorded. (f) The coherence function $g^{(1)}(\Delta\boldsymbol{r} = 2\boldsymbol{r}) = \langle\Psi^*(\boldsymbol{r}, L)\Psi(-\boldsymbol{r}, L)\rangle$ is obtained by computing the ensemble average of the measured contrast over 2000 realizations. (g) Experimental $g^{(1)}$ functions at $z=0$ and $z=L$. Light colored data are raw signals, and thin solid curves their azimuthal average. Thick solid curves are the final measurements, obtained by subtracting laser background imperfections. The light-cone position at $2c_s L$ is indicated by the red arrow. The dashed line is a guide emphasizing the algebraic decay.

interaction constant g, measured independently with the method of [42], and is shown with a red arrow on Fig. 1(g). Its position agrees with the observed onset of the $g^{(1)}$ plateau at a propagation length L.

The above observations fall in line with the common picture of prethermalization, where the short-time dynamics is governed by collisionless quasiparticles (here phonons) of long lifetime. To confirm this, following [15], we show in [46] from a low-energy Bogoliubov approach that the postquench dynamics starting from $\Psi(\boldsymbol{r}, z=0)$ can indeed be described in terms of long-lived phonons, giving the following scaling laws for the coherence function at the cell exit plane when $\epsilon \ll 1$:

$$g^{(1)}(\Delta\boldsymbol{r}) \propto I_t(r) \begin{cases} (\xi/\Delta r)^{\alpha} & \text{for } \Delta r < 2c_s L \\ \text{const} & \text{for } \Delta r > 2c_s L, \end{cases} \tag{2}$$

where $\xi = 1/\sqrt{4k_0 g I_0}$ is the healing length. The algebraic exponent is given by

$$\alpha \propto \epsilon^2 \Phi_{\mathrm{NL}} \sigma^2, \tag{3}$$

where $\Phi_{\mathrm{NL}} = g I_0 L$ is the nonlinear phase accumulated by light upon propagating through the vapor and σ is the speckle correlation length. Equations (2) and (3) emphasize another property of the prethermal state: within the light cone, the fluid of light resembles a 2D superfluid at thermal equilibrium, characterized by a quasi-long-range order [36–41]. This is not a strict thermal equilibrium though, since the characteristic dependence of α differs from the one of a thermal superfluid, for which $\alpha \propto \epsilon^2/\sigma^2$ and varies weakly with the interaction (see Ref. [46]).

To confirm the above scenario, we confront the experimental results to the prediction (3). We first show in Fig. 2(a) experimental coherence functions obtained for increasing values of the interaction $g \propto \Phi_{\mathrm{NL}}$ at fixed ϵ, σ. Algebraic exponents extracted from fits of $g^{(1)}(\Delta\boldsymbol{r})$ in the central region are shown in Fig. 2(b), and confirm the linear scaling of α with the interaction strength predicted by Eq. (3), at least for fluctuations ϵ^2 below 10%. The solid lines in Fig. 2(b) are linear fits to Eq. (3) and are used to set the proportionality coefficient of Eq. (3), such that there are no adjustable parameters for all other results presented in this Letter. A second set of measurements, using two different correlation lengths σ, is presented in Fig. 2(c). By normalizing the exponent $\alpha(\Phi_{\mathrm{NL}})$ to σ^2, we observe that $\alpha(\Phi_{\mathrm{NL}})/\sigma^2$ measured at different σ is independent of σ, which is again in agreement with the scaling law of Eq. (3).

Additionally, we investigated the behavior of $g^{(1)}$ with ϵ, setting $\sigma = 35\ \mu$m and the interaction strength $\Phi_{\mathrm{NL}} = 20$ rad to a relatively weak value. The results are reported in Fig. 3. We observe an increase of the algebraic exponent α with ϵ. A comparison of the extracted $\alpha(\epsilon)$ with Eq. (3) is shown in the inset. We find that the theory provides a good

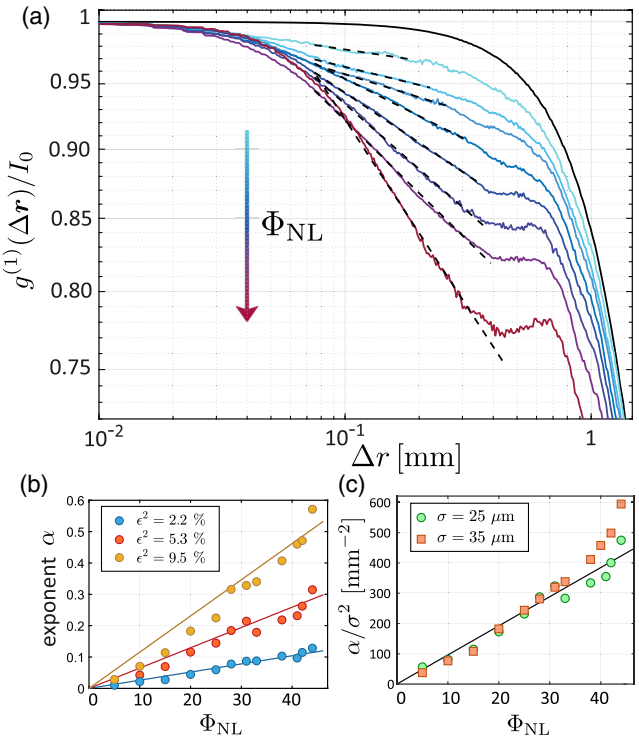

FIG. 2. Direct observation of the long-range algebraic order in a 2D prethermal state of light. (a) Normalized coherence function $g^{(1)}(\Delta\boldsymbol{r})/I_0$ vs Δr for increasing values of the nonlinear phase Φ_{NL}, at fixed $\sigma = 25\ \mu$m and $\epsilon^2 = 2.2\%$. Notice the double logarithmic scale. Dashed curves are algebraic fits to $1/\Delta r^{\alpha}$ in the central region. (b) Extracted algebraic exponents α vs Φ_{NL}, for three fluctuation strengths (blue: $\epsilon^2 = 2.2\%$, red: $\epsilon^2 = 5.3\%$, yellow: $\epsilon^2 = 9.5\%$) (c) Algebraic exponent α/σ^2 at fixed $\epsilon^2 = 5.3\%$, for two different speckle correlation lengths σ (green circles: $\sigma = 25\ \mu$m, red squares: $\sigma = 35\ \mu$m). All data points fall on the same curve, confirming the scaling $\alpha \propto \sigma^2$. Solid curves are linear fits to Eq. (3).

description of experimental results, as long as the initial fluctuations remain small, typically $\epsilon^2 \lesssim 0.25$. In fact, deviations at higher ϵ values should not come as a surprise, since the characteristic algebraic, "low-energy" behavior (2) of the coherence function is expected to hold for $\epsilon \ll 1$ only. Indeed, in our 2D system the emergence of a prethermal state can be seen as a consequence of a weak breaking of translation invariance stemming from the initial speckle, a scenario for prethermalization put forward in [11].

To further characterize the nonequilibrium dynamics, we have studied the evolution of $g^{(1)}(\Delta\boldsymbol{r})$ up to larger values of ϵ, setting a stronger interaction strength $\Phi_{\mathrm{NL}} = 44$ rad. From the above discussion, one could naively expect that, upon increasing ϵ, the low-energy state (2) leaves room for a nonuniversal dynamics, where no prethermalization stage arises and where $g^{(1)}(\Delta\boldsymbol{r})$ has no specific structure. Instead, we have experimentally observed that the coherence function smoothly turns from algebraic to exponential as ϵ is increased:

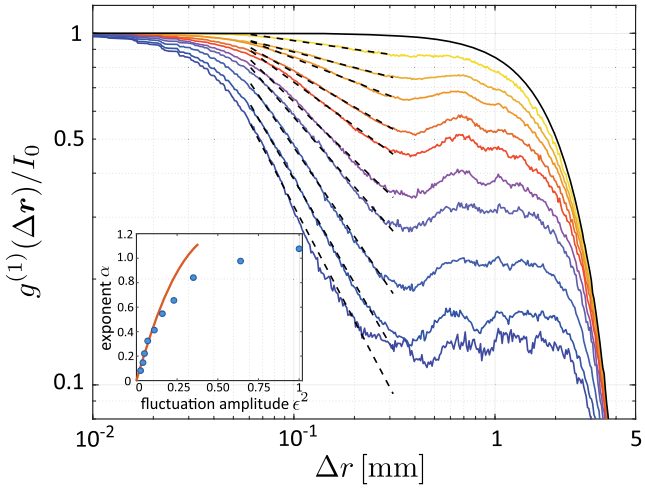

FIG. 3. Impact of the initial fluctuation amplitude ϵ on the prethermal state. Normalized coherence function $g^{(1)}(\Delta \boldsymbol{r})/I_0$ vs Δr for increasing values of the fluctuation amplitude ϵ ($\epsilon^2 = 0$, 0.027, 0.04, 0.05, 0.07, 0.11, 0.16, 0.23, 0.35 from top to bottom). Here $\sigma = 35$ μm and $\Phi_{\rm NL} = 20$ rad. Dashed curves are algebraic fits to $1/\Delta r^{\alpha}$ in the central region. The inset shows the extracted algebraic exponents (blue dots), together with the prediction (3) [with no free parameters since we set the proportionality coefficient with that of Fig. 2].

$$g^{(1)}(\Delta \boldsymbol{r}) \sim \exp(-\Delta r/r_c). \tag{4}$$

The crossover from Eq. (2) to this exponential behavior is presented in the measurements of Fig. 4(a). We have confirmed it by a computation of the sum of the squared estimate of errors (SSEs) that measures the discrepancy between the $g^{(1)}$ data and a fit to either Eq. (2) or (4), see Fig. 4(b). Note that such an exponential decay differs from the Gaussian correlations of the initial speckle and, in that, is associated with a genuine new dynamical regime emerging from the quench. Another statistical test, coefficient of determination R^2, is presented in [46] and confirms the transition at $\epsilon^2 \sim 0.2$. We also observed this crossover in *ab initio* numerical simulations presented in [46], and found it to be a generic feature of $g^{(1)}$ in the prethermal regime as ϵ or $\Phi_{\rm NL}$ is increased to moderate values. This phenomenon was also previously pointed out in [15]. At a physical level, this algebraic-to-exponential crossover is reminiscent of the celebrated Kosterlitz-Thouless (KT) transition, which drives 2D Bose gases at thermal equilibrium from a superfluid to a normal-fluid state when the temperature is raised. Although out of equilibrium, our fluid of light displays a similar behavior in the prethermal regime. This unexpected phenomenon can be understood by noticing that, at low ϵ and/or small interaction strength, the energy injected into the system during the quench is low, and so is the effective prethermalization "temperature." This results in a prethermal state with quasi-long-range order, which can be seen as the

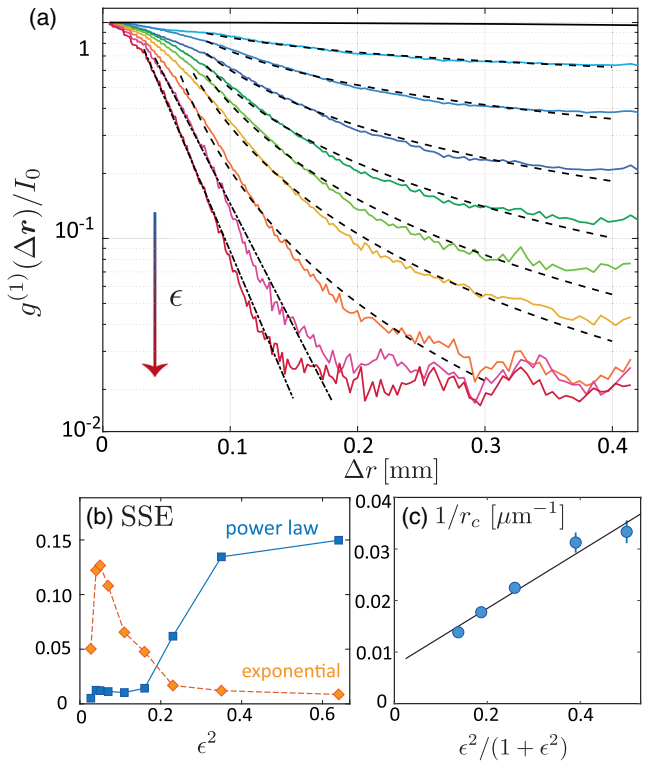

FIG. 4. Crossover from algebraic to short-range (exponential) correlations at stronger fluctuation amplitudes, with the interaction strength set to a value $\Phi_{\rm NL} = 44$ rad, twice larger than in the previous data (notice the log scale). (a) Normalized coherence function $g^{(1)}(\Delta \boldsymbol{r})/I_0$ vs Δr when increasing more significantly ϵ ($\epsilon^2 = 0$, 0.027, 0.04, 0.05, 0.07, 0.11, 0.16, 0.23, 0.35, 0.64 from top to bottom). Here $\sigma = 35$ μm is fixed. (b) Sum-squared error between experimental data and exponential (orange) vs algebraic (blue) fit. For $\epsilon^2 > 0.35$ the exponential fit becomes more accurate than the algebraic fit. (c) Rate $1/r_c$ of the exponential decay, see Eq. (4), vs the fluctuation amplitude. The linear fit (black) confirms the theoretical scaling of Eq. (5).

dynamical counterpart of a 2D, equilibrium superfluid at low temperature. When ϵ and/or g is increased, on the other hand, one reaches prethermal states of effectively larger temperature. The resulting fluid displays exponentially decaying correlations, analogous to the normal phase of a 2D Bose gas above the KT temperature. At the crossover between the two regimes, the exponent value is $\alpha = 1.7 \pm 0.1$ (see Ref. [46]), in strong contrast with a KT phase transition in thermal equilibrium homogeneous systems where the exponent is 0.25 [40].

To gain more insight on the prethermal regime of exponential correlations, we studied the dependence of the correlation length r_c of the exponential decay, see Eq. (4), on the initial fluctuation amplitude ϵ. To unveil this dependence, one can take advantage of total energy conservation $E_t = \int d\boldsymbol{r}[1/(2k)|\nabla\psi(\boldsymbol{r})|^2 + g/2|\psi(\boldsymbol{r})|^4]$ during the nonequilibrium evolution. Equating E_t to the normal state energy (4), we obtain

$$\frac{1}{r_c} \propto \frac{\epsilon^2}{1+\epsilon^2}. \tag{5}$$

We have also confirmed this law from extensive numerical simulations presented in [46]. Experimental values of $1/r_c$, extracted from our measurements of $g^{(1)}$, are shown in Fig. 4(c). When plotted vs $\epsilon^2/(1+\epsilon^2)$, they show a good agreement with the prediction (5).

In summary, our experimental description of a 2D fluid of light through a direct probe of its spatial coherence has revealed the dynamical emergence of algebraic prethermalization following an interaction quench. Unlike previous studies involving near-integrable systems in 1D, prethermalization emerges as a result of the weak breaking of translation invariance after the quench. Our results further point toward the existence of a crossover from algebraic to exponential correlations in the prethermal regime of 2D systems, an intriguing phenomenon that we interpret as a nonequilibrium precursor of the thermodynamic KT transition. This effect opens exciting perspectives for further studies of nonequilibrium quantum fluids. While a comprehensive description of 2D thermalization processes remains open, our analysis emphasizes the assets of photon fluids for probing the dynamics of far-from-equilibrium many-body systems.

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